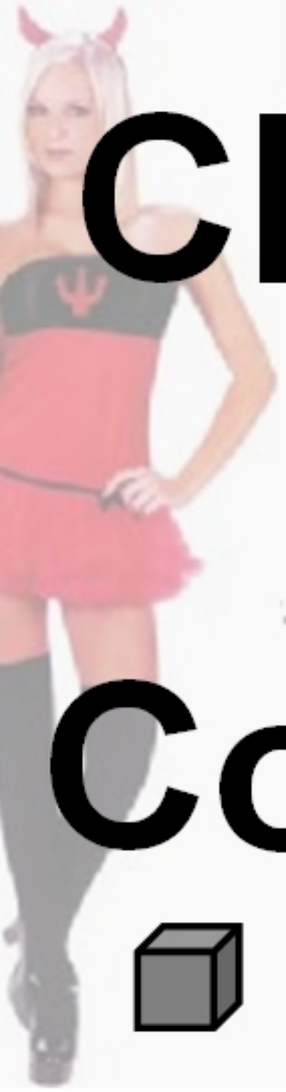
Scheduling of Independent Tasks



Clairvoyance or Cooperation?

Parallelisation of rendering algorithms

University of Paderborn

University of BRISTOL

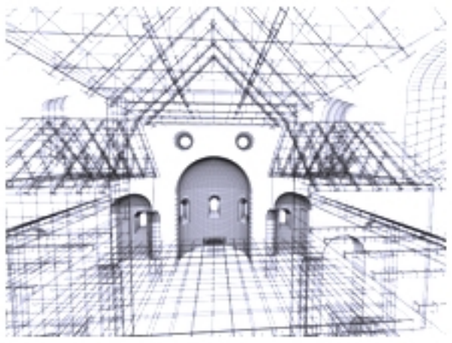
Comenius Uni Bratislava

Project HiQoS (Uni Paderborn et al): parallel ray tracing & radiosity

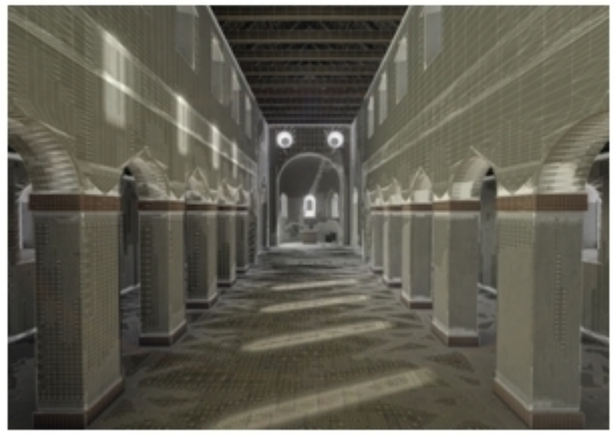
Ray tracing:
„method for everybody“

hour

Radiosity:
„method for professionals“



... many hours



sec

Project HiQoS (Uni Paderborn et al): parallel ray tracing & radiosity

Ray tracing:
„method for everybody“

minutes

Radiosity:
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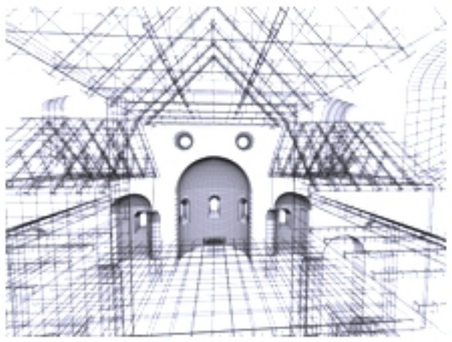
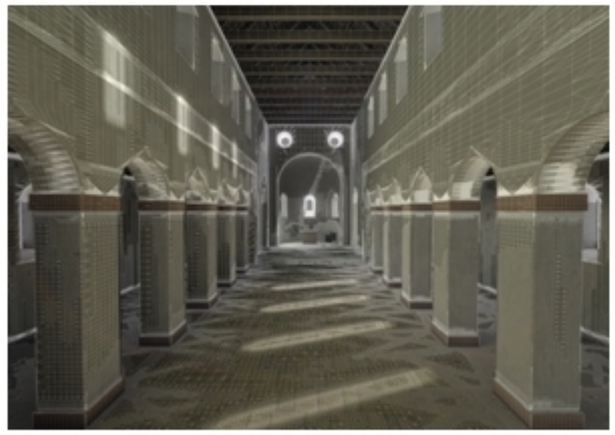


... many
minutes



sec

Background: parallel ray tracing (how to distribute the work?)

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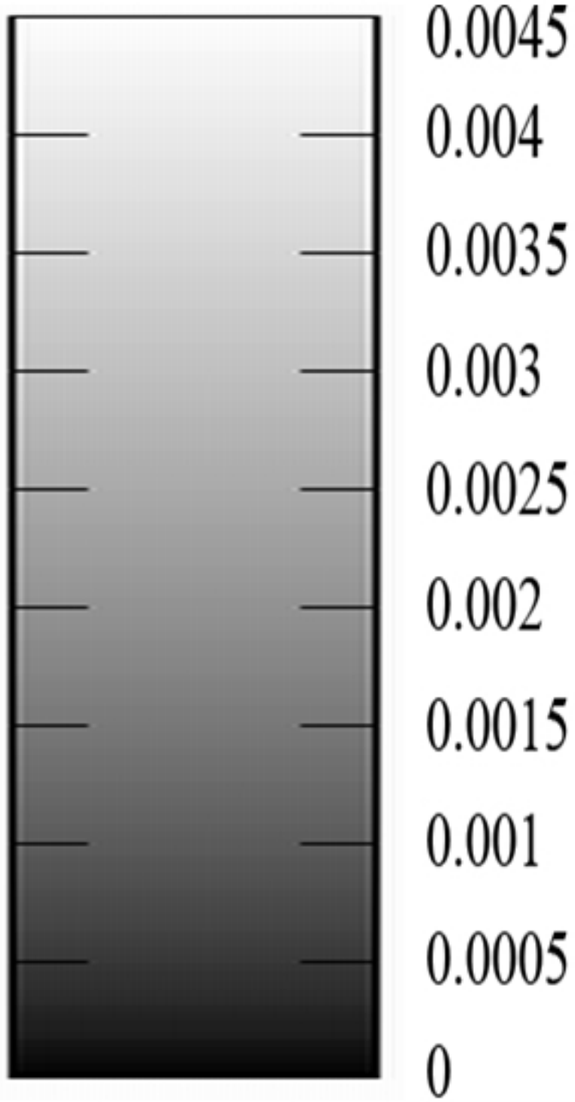
Computational times of pixels



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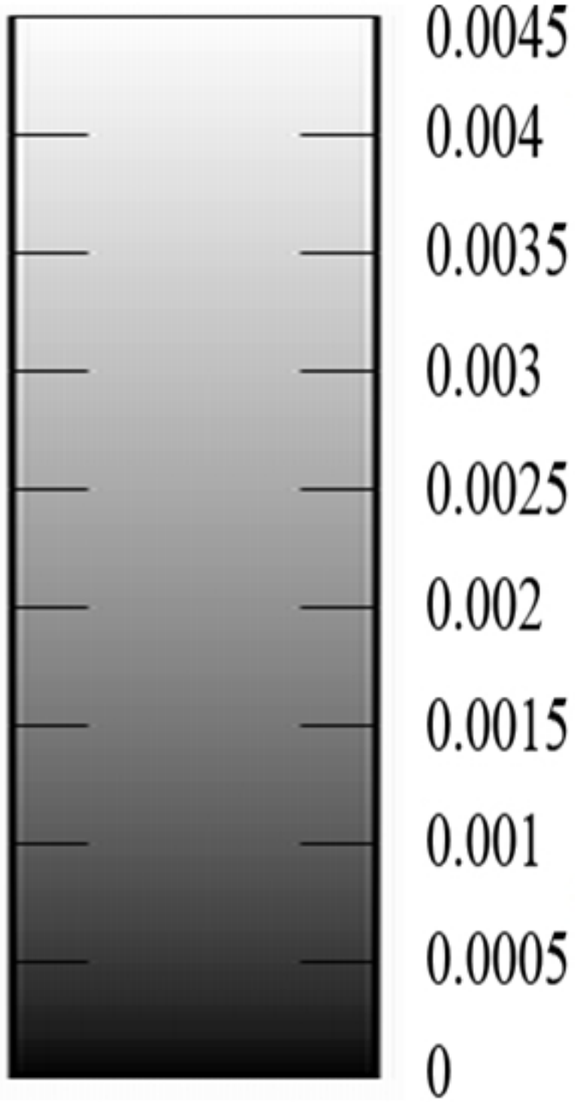
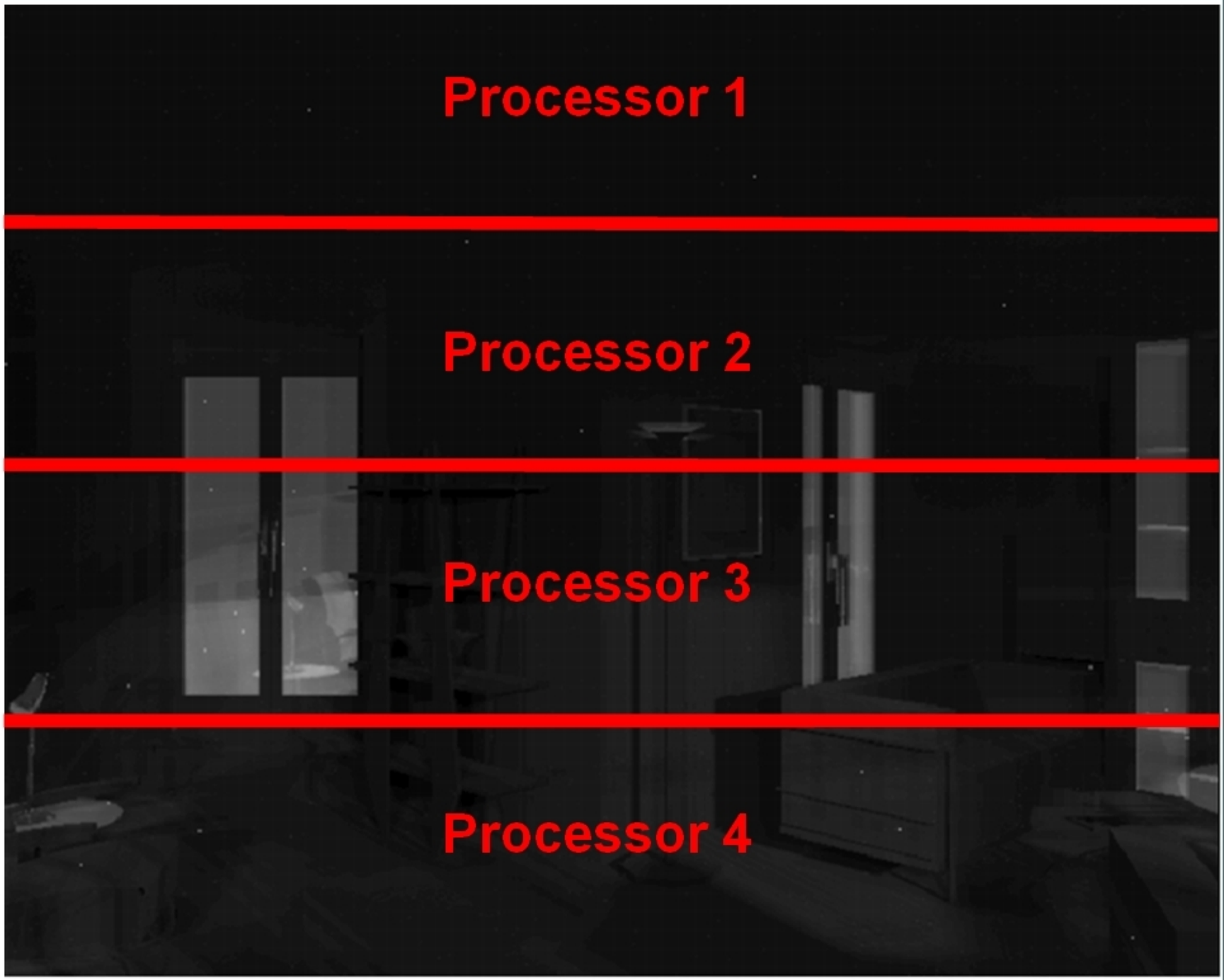




Although work distribution is equal, time is not

Clairvoyance vs. Cooperation in Scheduling of Independent Tasks

jail
(limits someone's freedom)

tasks
(differ only in contents)

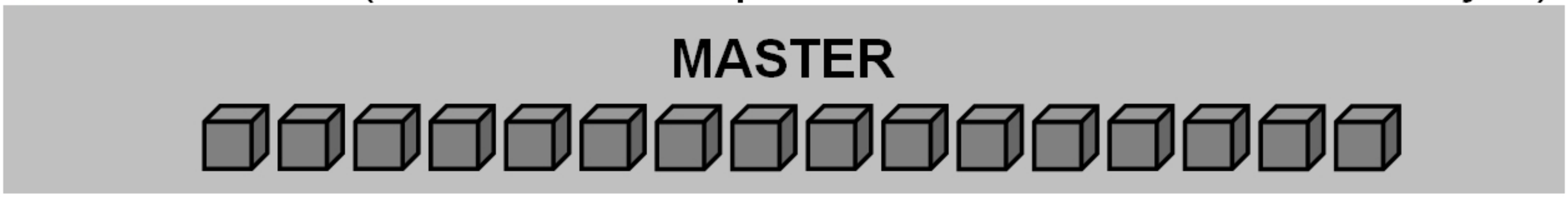
bad

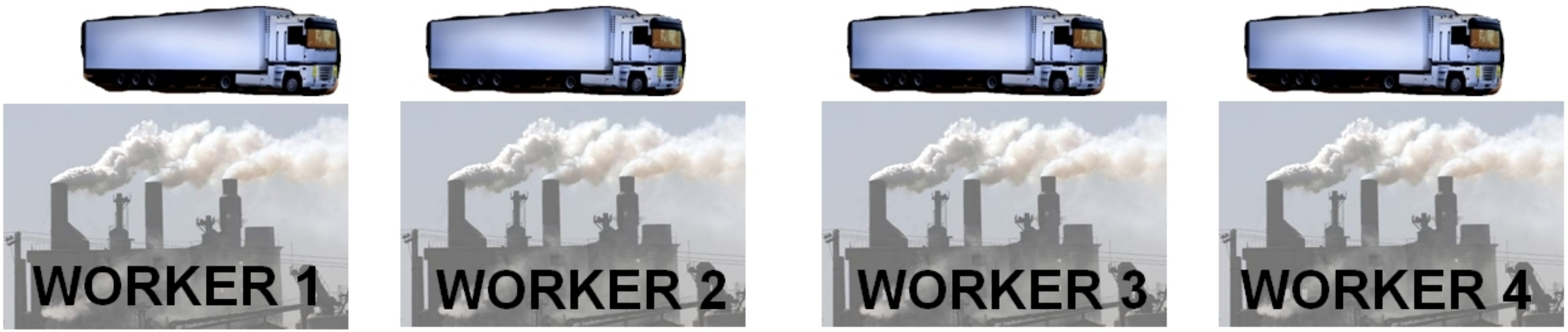
Introduction

W = 16 tasks

N = 4 workers

L = ? seconds (a constant independent on the nr of tasks in a job)





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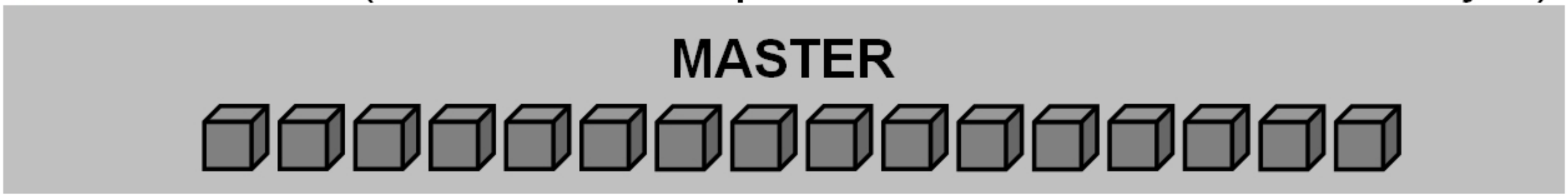

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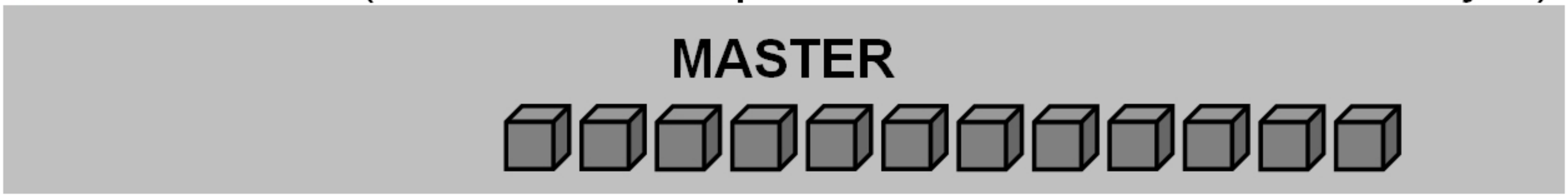

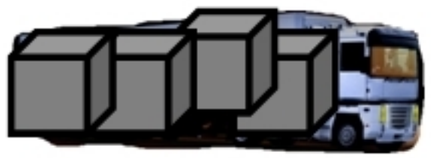

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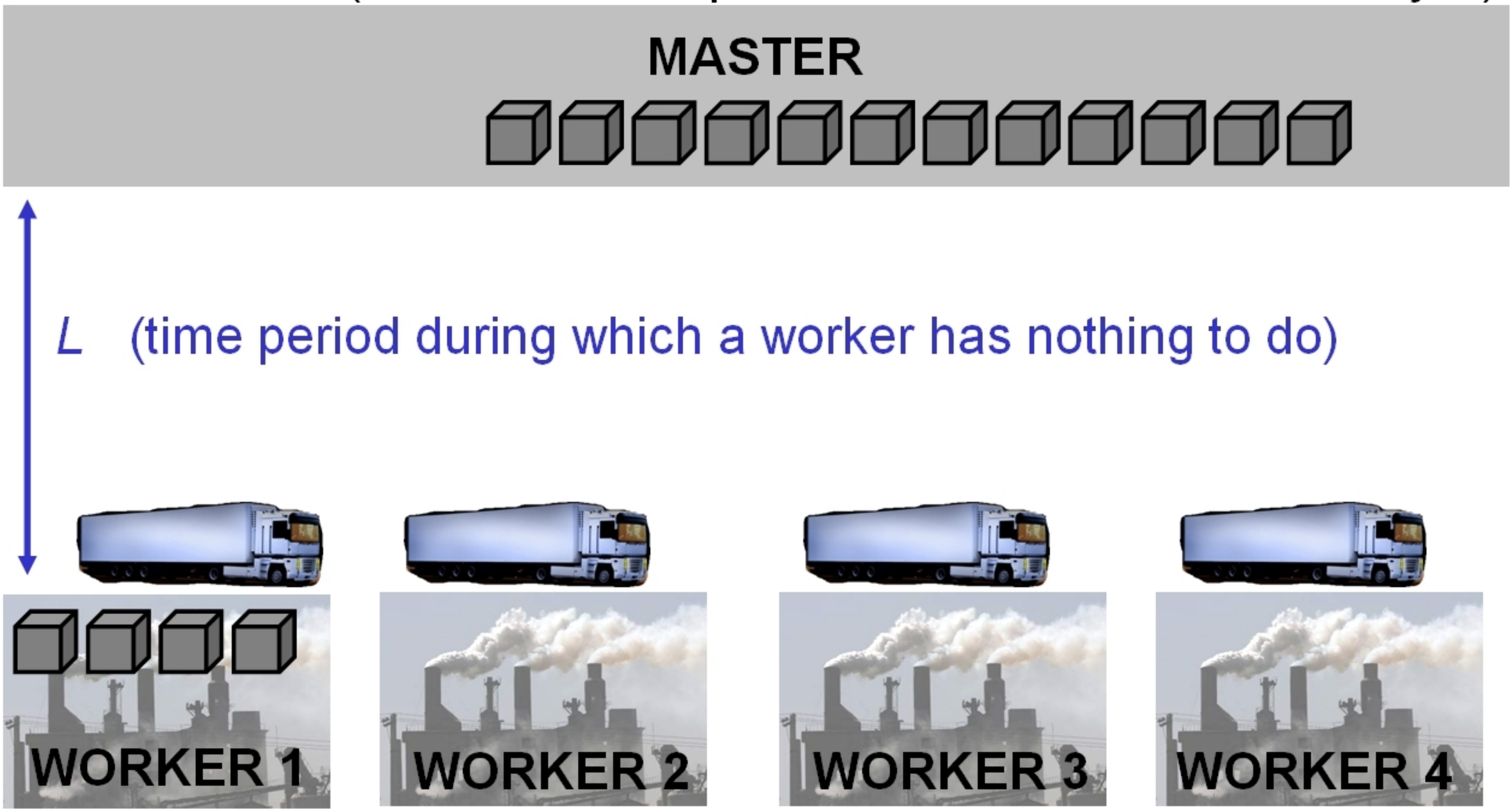

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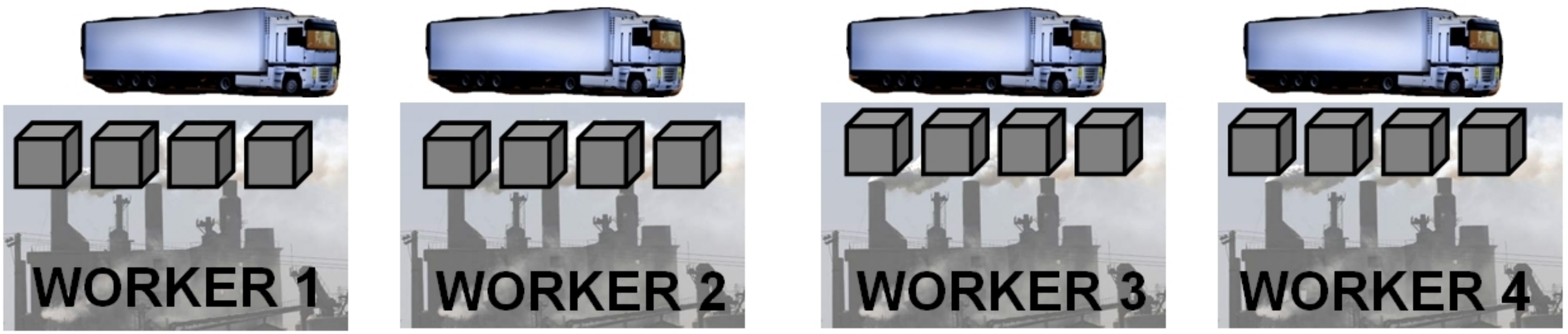
MASTER



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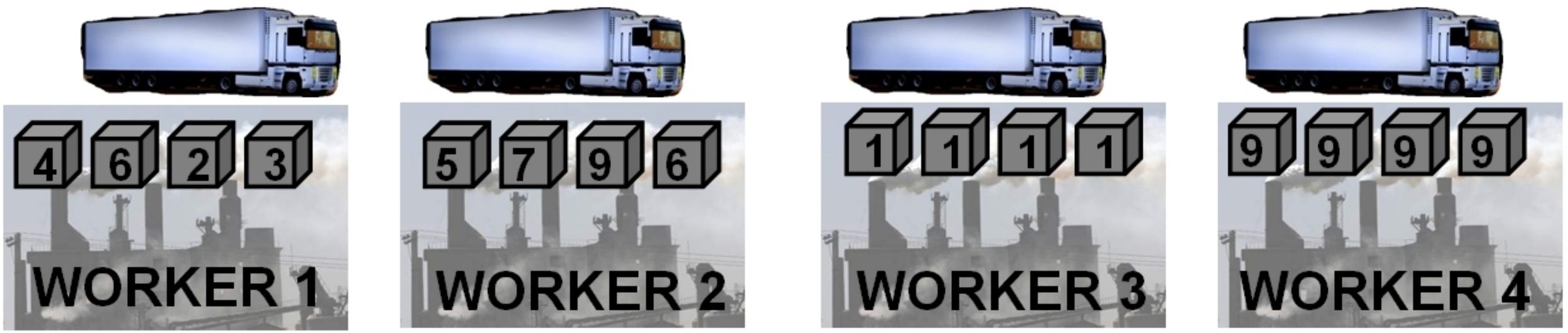
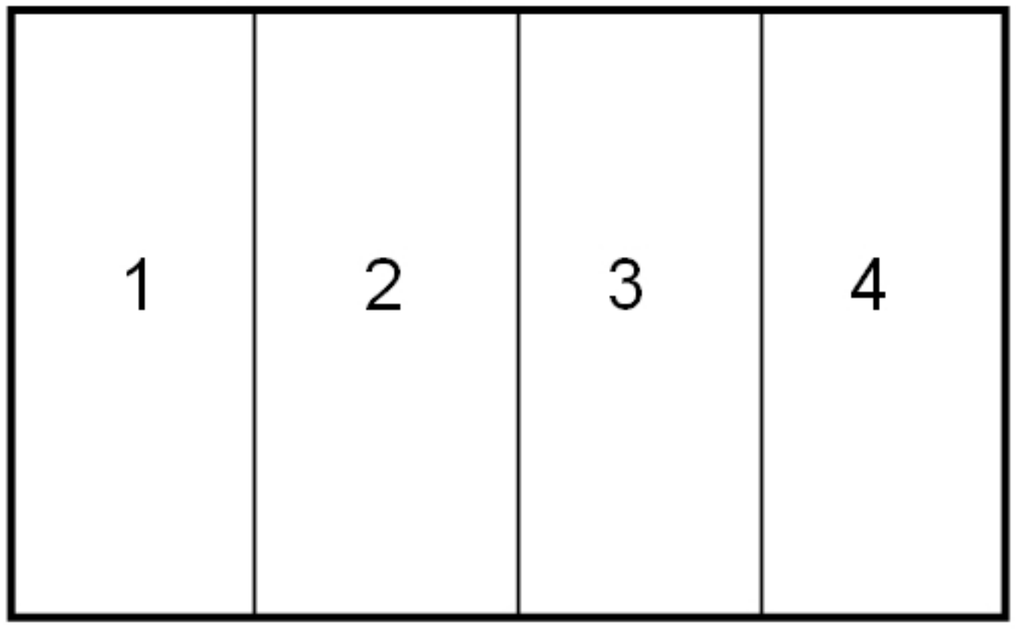
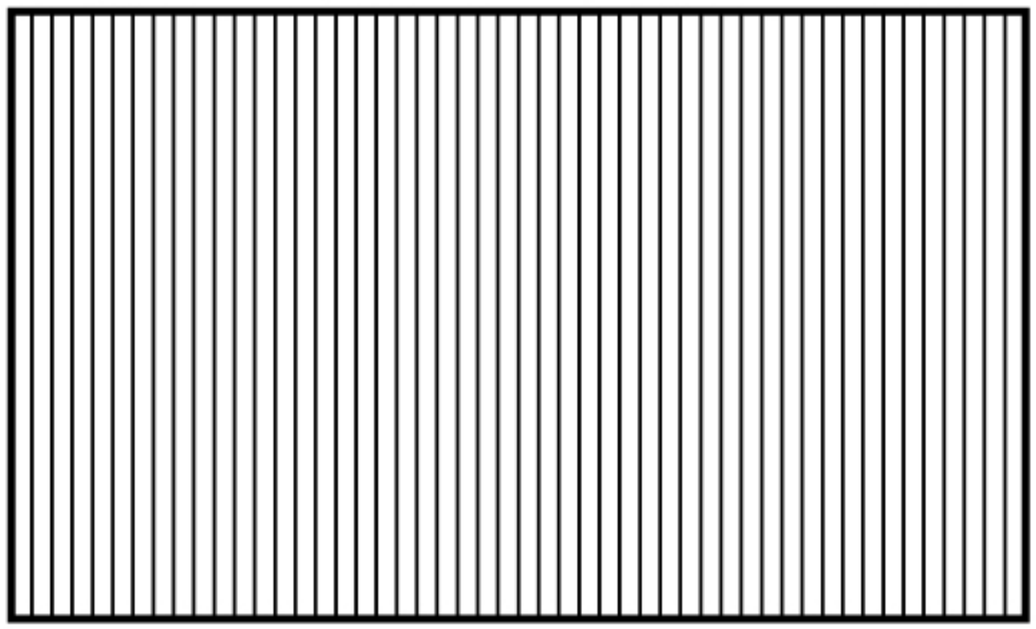


Chunking algorithm (goal: minimise makespan)

Static job assignment



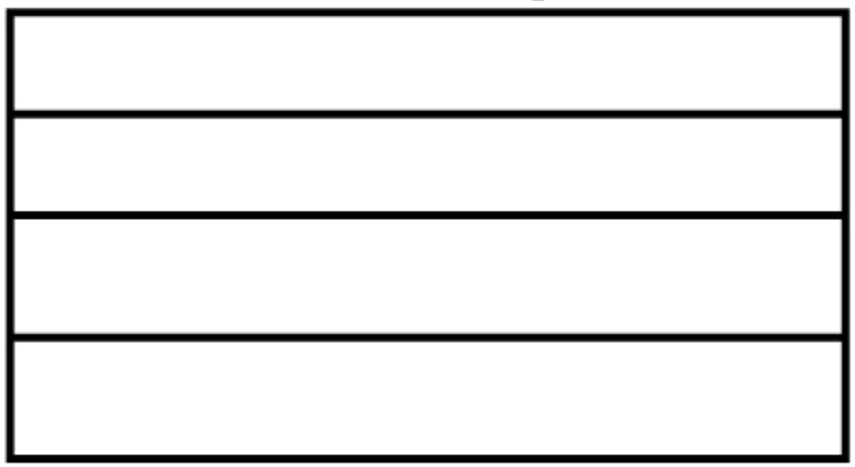
Dynamic job assignment



- almost no communication
- heavy imbalance

- heavy communication
- perfect balance (up to 1 task)

Job "shape" is irrelevant, only job size matters



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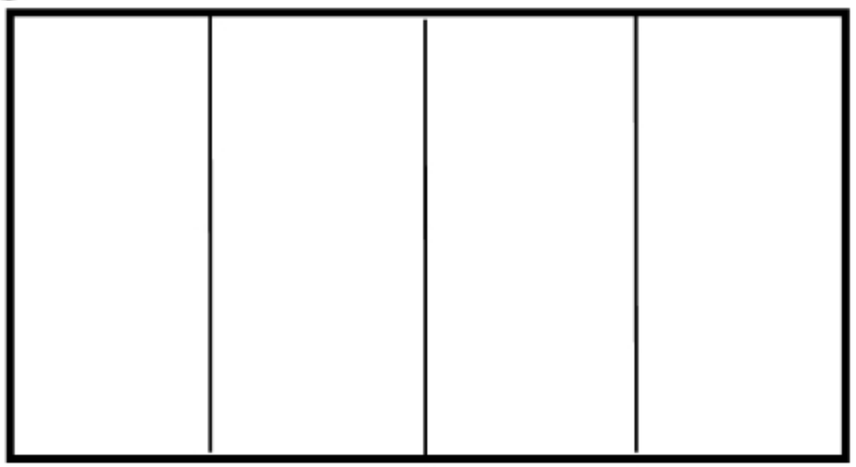


MASTER process

```
master(int W, int N)
{
  int K;
  int work = W;
  while (work > 0)
  {
    wait for a job request from an idle worker;
    compute job size K; /* K may be different for different requests */
    assign job of K tasks to the idle worker;
    work = work - K;
  }
  reply remaining job requests with NO_MORE_WORK;
}
```

***How many tasks* should the MASTER process assign in one *job*? Some practitioners suggest 16 tasks, other practitioners suggest 4096 tasks. Theoreticians produce even more suggestions. (Show must go on.)**

Problem definition

Unknown:
Tasks' durations t_1, t_2, …, t_W
(and also the latency L, if we trust theoreticians)

Given:

N	nr. of worker processes, all equally fast
W	nr. of tasks, independent on each other
L	latency (this is actually also unknown, but perhaps we trust network providers)

CLAIRVOYANCE: partial information on tasks' durations

Clairvoyant probabilistic model:
e.g. tasks' durations are generated by $N(\mu, \delta)$ and μ, δ are a-priori known

Goal:
Minimise expected time

Clairvoyant deterministic model:
e.g. maximal task's duration T_{max} is a-priori known,
or $T = T_{max} / T_{min}$ is a-priori known
or…

Goal:
Minimise maximal time (makespan)

Clairvoyant deterministic model

Unknown:

Tasks' durations t_1, t_2, …, t_W

Given:

N	nr. of worker processes, all equally fast
W	nr. of tasks, independent on each other
L	latency (assignment of one job)

and some clairvoyant information on tasks' durations,
e.g. $T_{max} = \max(t_1,\ldots,t_W)$

Goal:

Minimise maximal time ("worst case"). Mmm, what is "worst case"?

Chunking algorithm (goal: minimise maximal makespan)

Given:

N nr. of workers

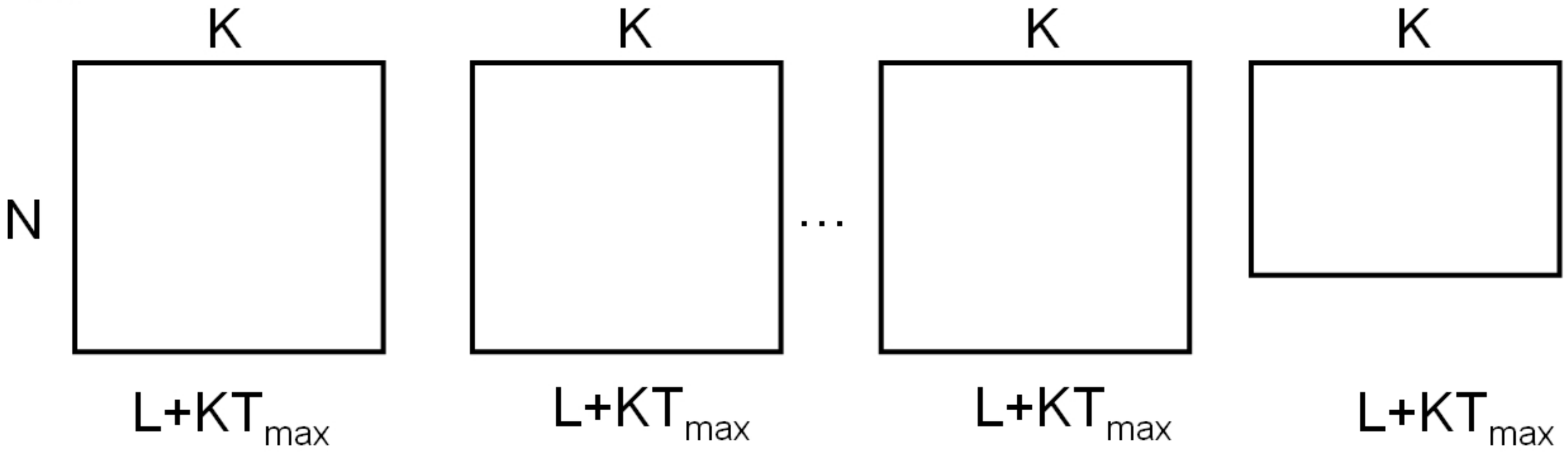
W nr. of tasks

L latency

T_{max} max. task complexity

Unknown:

K_{opt} (chunk size)



$$M = \left(1 + \frac{W}{NK}\right)(L + KT_{\max}) \qquad K_{opt} = \sqrt{\frac{WL}{NT_{\max}}} \rightarrow M_{opt} = \frac{WT_{\max}}{N} + L + 2\sqrt{\frac{WT_{\max}L}{N}}$$

$$M'(K) := 0$$

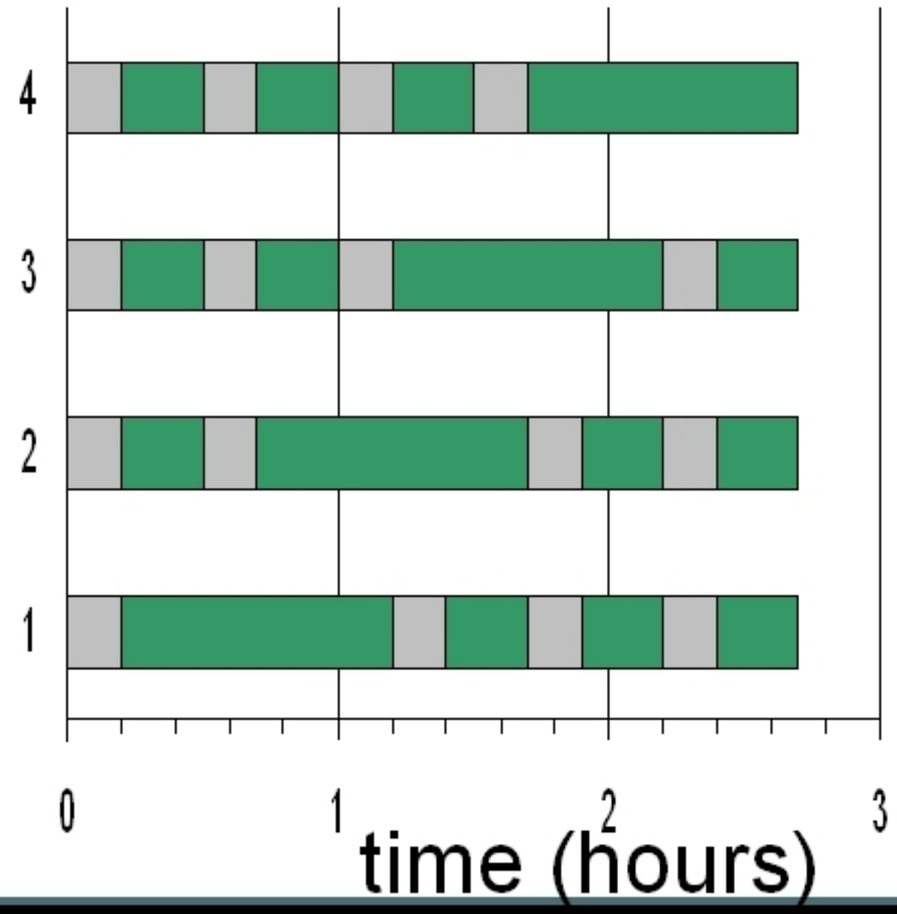
Minimise maximal time? Worst case depends on input!

Goal:

Minimise maximal time (in “worst case”)

Example: $N = 4$, $W = 16$, $L = 12$ min, $T_{max} = 1$ hour. Let us use chunking algorithm which always assigns jobs of the same size K. The “worst case” is that all W tasks are of duration $T_{max} = 1$ hour (indeed, this maximises the time). We want to set K so that we minimise the makespan (total time)

“K=4 is the best choice”



“Come on. What if only 4 tasks are long? K=1 is a better choice”

Clairvoyant deterministic model, revised

Unknown:
Tasks' durations $t_1, t_2, \ldots, t_W$

Given:

N	nr. of worker processes, all equally fast
W	nr. of tasks, independent on each other
L	latency (assignment of one job)

and some clairvoyant information on tasks' durations, e.g. T_{max}

Goal:
Minimise competitive ratio 

Definition. **Competitive ratio $CR_{S(A)}(W, N, L)$** of algorithm S with a-priori information A is the maximal ratio between the makespan of algorithm S and the makespan of the best offline algorithm—over all sequences $t_1, \ldots, t_W$ which conform to the a-priori information A:

$$\mathcal{CR}_{S(A)}(W, N, L) = \sup_{t_1 \ldots t_W} \frac{\mathcal{M}_{S(A)}(t_1 \ldots t_W)}{\mathcal{M}_{\text{best\_offline}}(t_1 \ldots t_W)}$$

Definition. An algorithm with a **smaller competitive ratio is better** than an algorithm with a higher competitive ratio

Notes on competitiveness

$$\mathcal{CR}_{S(A)}(W,N,L) = \sup_{t_1 \ldots t_W} \frac{\mathcal{M}_{S(A)}(t_1 \ldots t_W)}{\mathcal{M}_{\text{best\_offline}}(t_1 \ldots t_W)}$$

An offline algorithm knows everything a-priori (before the computation starts)

Finding the shortest schedule offline is an NP-hard problem, so its computation may take a long time when W is large—but this time does not add to $M_{best\_offline(t_1, \ldots, t_W)}$

However, $M_{best\_offline(t_1, \ldots, t_W)}$ includes one-time penalty L, as the pre-computed schedule is assigned to workers once (in parallel)

$CR_{S(A)} \geq 1$ for any algorithm S. Makespan of S also includes at least one assignment penalty L. Moreover, if S only has a partial a-priori information A on durations $t_1, t_2, \ldots, t_W$, it cannot generally produce the shortest schedule

$CR_{S(A)}$ is a function of W, N, L. It is particularly interesting to observe its limit behaviour for $W \to \infty$. This indicates how good the algorithm S is

Chunking algorithm (goal: minimise competitive ratio)

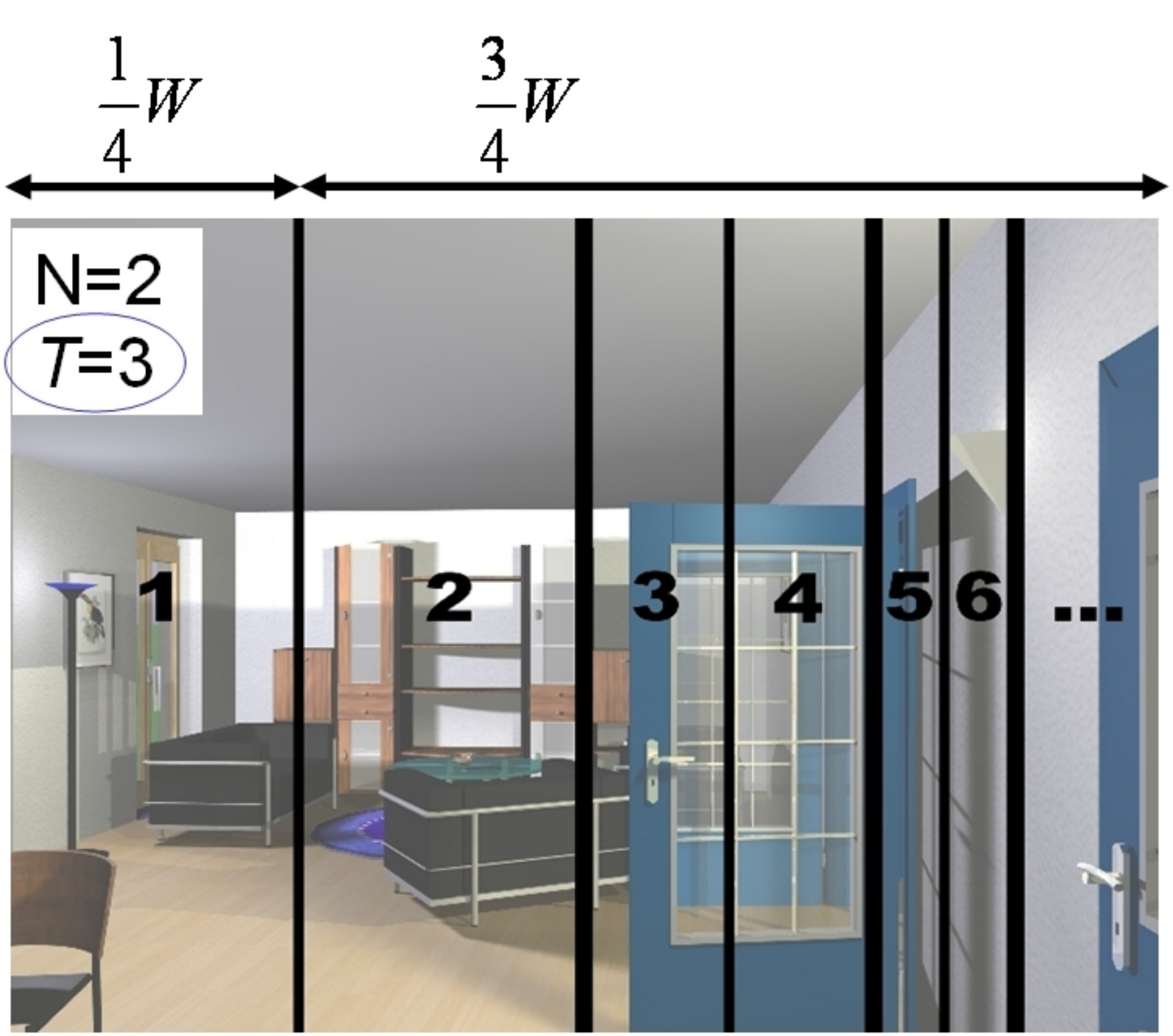
Theorem 1. *For all* W, N, L, T_{max} *such that* $W > N^3 + N^2(N-1)T_{max}/L$ *competitive ratio of an arbitrary assignment algorithm with no work redistribution and with a-priori knowledge of* T_{max} *is at least* N *(i.e.* $\Omega(N)$ *for* $W \to \infty$*).*

👎 CR of chunking increases linearly with an increasing number of workers for $W \to \infty$
(*W* does not actually have to be very large, $W \cong N^3$ will do)

👎 **This holds not only for every setting of *K*, but for all algorithms which know only *W*, *N*, *L* and T_{max} in advance**

👎 **A-priori knowledge of T_{max} does not help much, efficiency of parallel processing goes down the drain**

Factoring algorithm, example

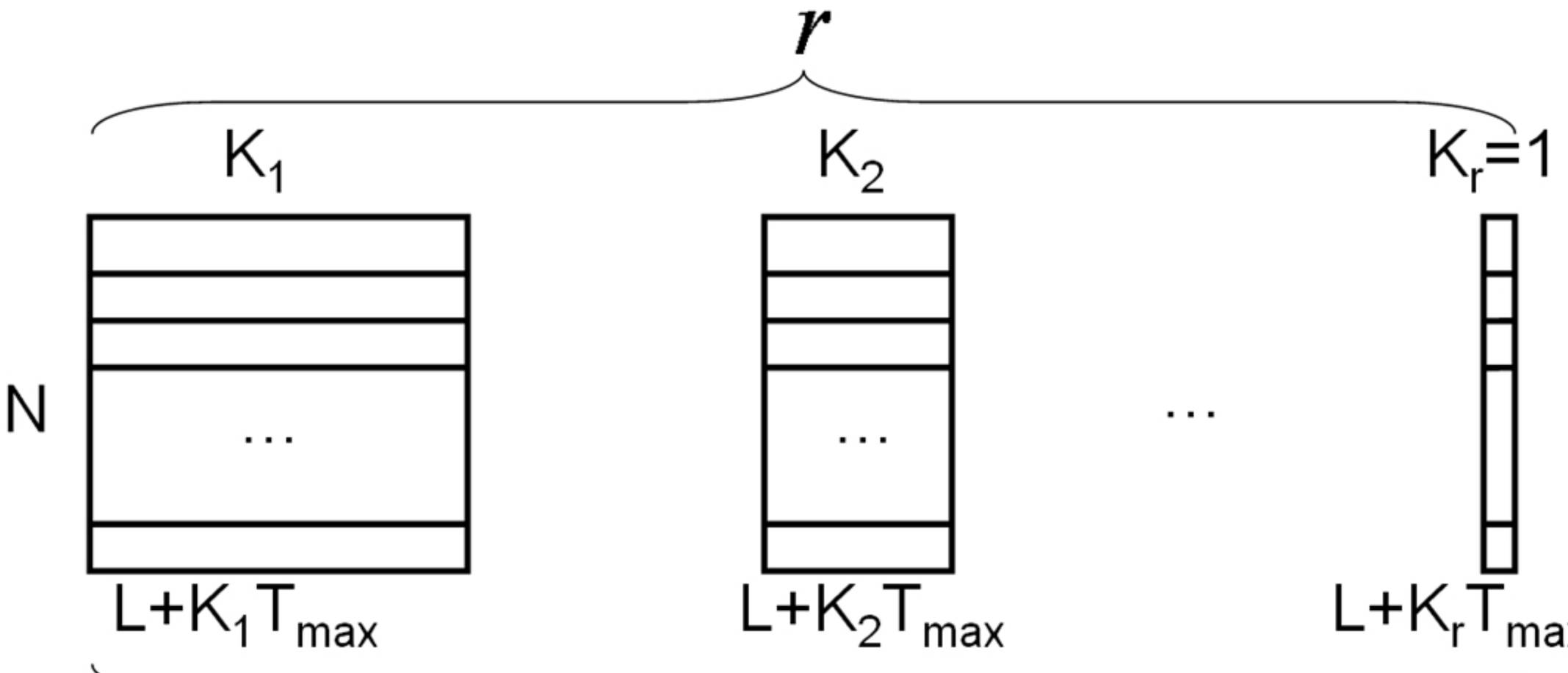


N nr. of workers
W nr. of tasks
T max. ratio of tasks' durations, $T = T_{max} / T_{min}$
Unknown: the job size K_i used in round i

👍 Perfect balance
(max. imbalance is 1 task)

$$K_i = \max\left(1, \left\lfloor \frac{w_i}{1 + T \cdot (N-1)} \right\rfloor\right)$$

Factoring algorithm (goal: minimise competitive ratio)



$$\frac{W}{N}\,T_{\max} + Lr$$

$$K_i = \max\left(1, \left\lfloor \frac{w_i}{1 + T \cdot (N-1)} \right\rfloor\right)$$

This yields $r = \dfrac{\ln(W/N)}{\ln \dfrac{1+T(N-1)}{(N-1)(T-1)}}$

Factoring (goal: minimise competitive ratio)

Theorem 2. *For all N, L, T competitive ratio of the factoring algorithm with a-priori knowledge of T using factor $F = 1 + T(N-1)$ is $1+O((\ln W)/W)$ and approaches 1 for $W \to \infty$.*

👍 This means that the knowledge of $T = T_{max} / T_{min}$ is more valuable than the knowledge of T_{max}

👎 T must be known before it all starts. If T is underestimated or overestimated, then this theorem does not hold

$T \to 1.0 \equiv$ chunking with K = W / N

$T \to \infty \equiv$ chunking with K = 1

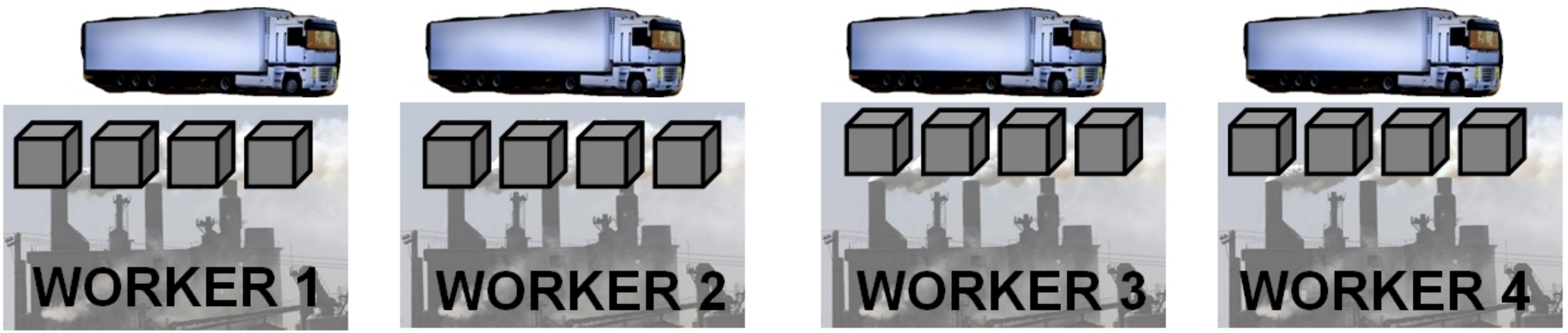
Work stealing algorithm, example

W = 16 tasks

N = 4 workers

L = ? seconds

MASTER



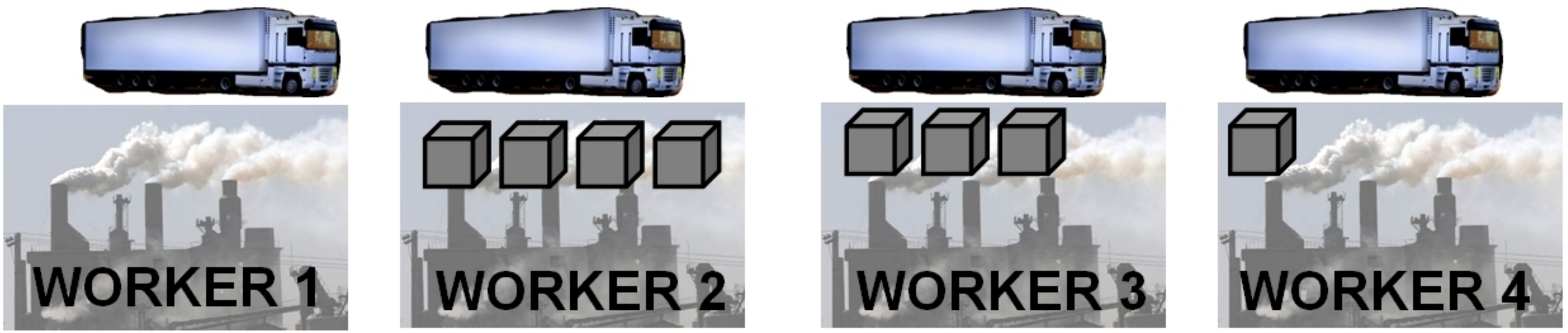
Work stealing algorithm, example

W = 16 tasks

N = 4 workers

L = ? seconds

MASTER



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MASTER

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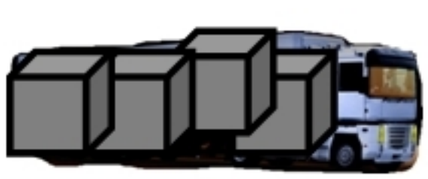

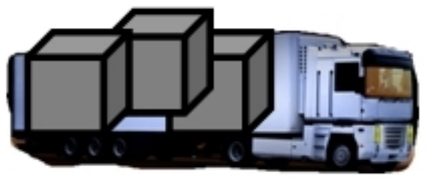

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MASTER

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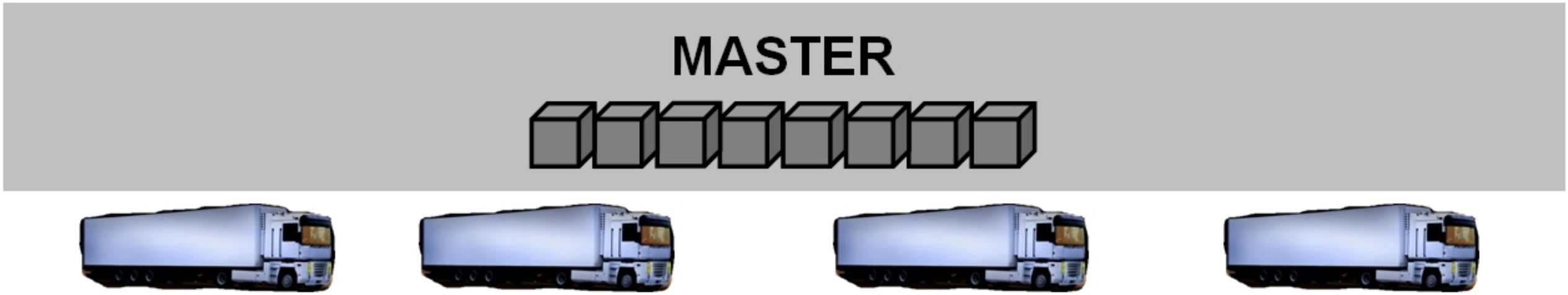


Work stealing algorithm, example

W = 16 tasks

N = 4 workers

L = ? seconds

MASTER

Work stealing algorithm, example

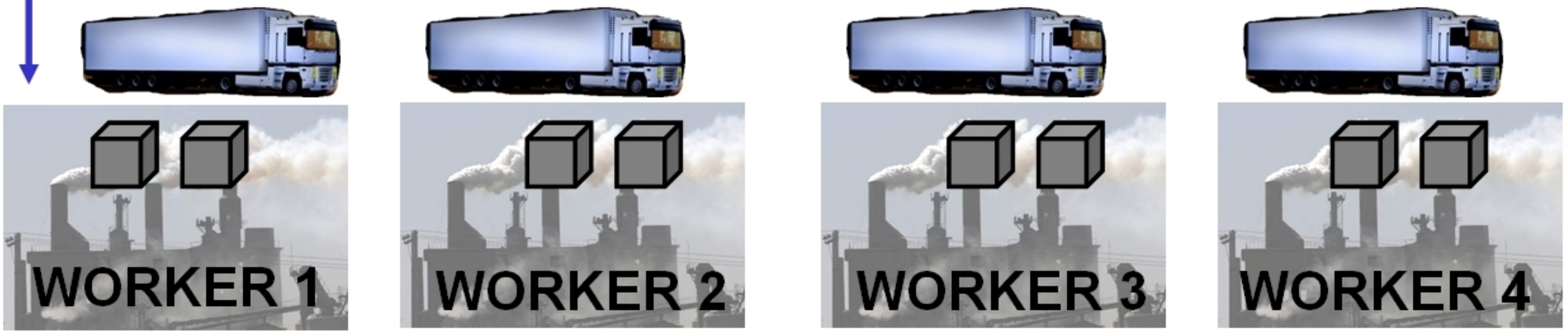
W = 16 tasks

N = 4 workers

L = ? seconds

MASTER

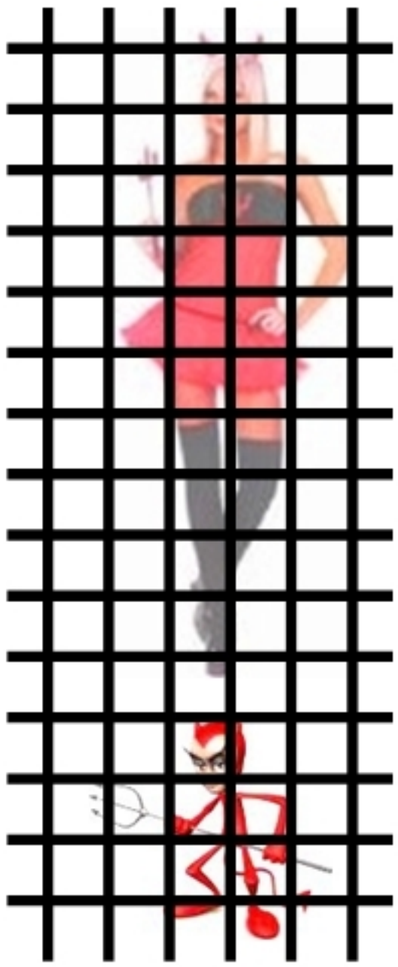
L' (time period during which a worker has nothing to do)



Work stealing algorithm (goal: minimise competitive ratio)

Theorem 3. *For all N, L, L' competitive ratio of the work stealing algorithm with no a-priori knowledge is $1 + O((\ln W)/W)$ and approaches 1 for $W \to \infty$.*

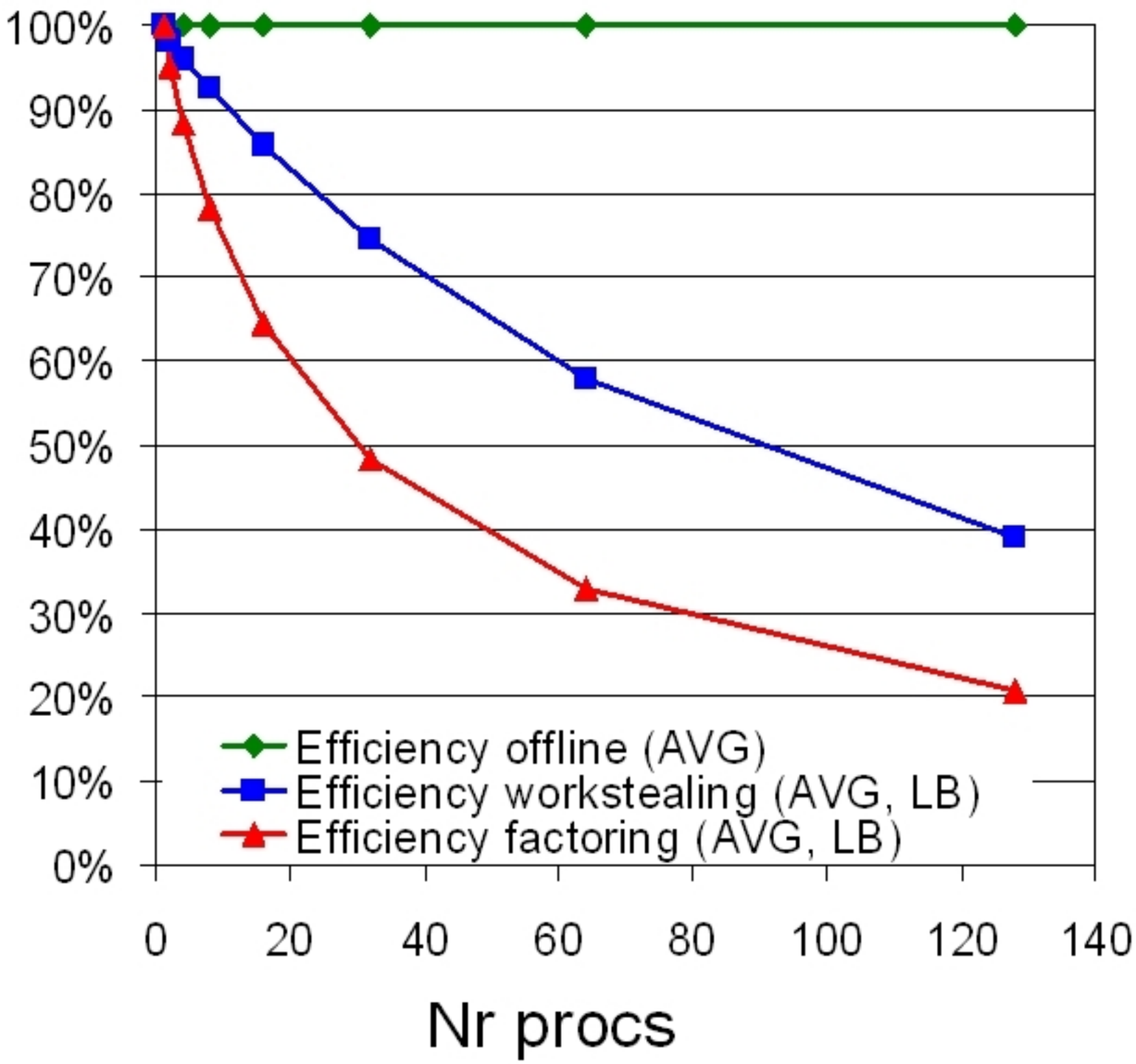
👍 Work stealing has the same asymptotic behaviour as factoring

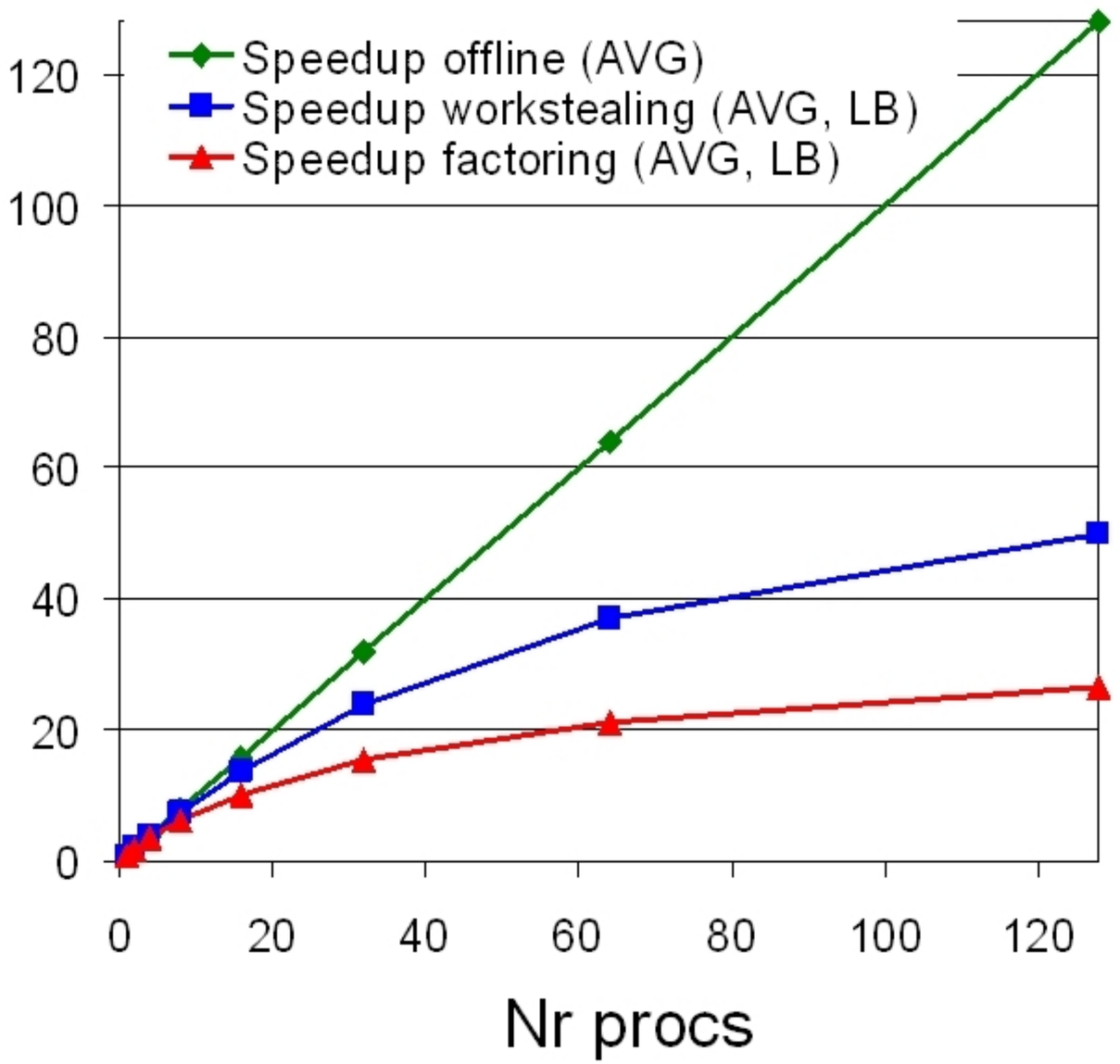


👍 Work stealing needs
NO CLAIRVOYANCE
(i.e. no a-priori information)

Practical consequence: efficient use of resources

L = 0,003 sec
Tmin = 0,01 sec
Tmax = 10 sec
W = 100000





Clairvoyance or Cooperation?

Comparison of 3 online algorithms:

- Chunking, requires knowledge of T_{max}

$lim_{W\to\infty}\ CR = N$, where N is number of workers (i.e. processors)

- Factoring, requires knowledge of T_{max} / T_{min}

$lim_{W\to\infty}\ CR = 1$

- Work stealing, requires no knowledge

$lim_{W\to\infty}\ CR = 1$

Work stealing is a clear winner even in a direct comparison which is not asymptotic—if certain conditions hold (they hold "in practice-relevant settings"). Additionally, work stealing offers attractive extensions: dynamic task creation, fault-tolerance, … Most importantly, **work stealing requires no parameters**

Clairvoyance or Cooperation?